

Supersymmetric IIB Matrix Models from Space-time Uncertainty Principle and Topological Symmetry

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Abstract

Starting with topological field theory, we derive space-time uncertainty relation proposed by Yoneya through breakdown of topological symmetry in the large N matrix model. Next, on the basis of only two basic principles, those are, generalized space-time uncertainty principle containing spinor field and topological symmetry, we construct a new matrix model. If we furthermore impose a requirement of $N = 2$ supersymmetry, this new matrix model exactly reduces to the IKKT model or the Yoneya model for IIB superstring depending on an appropriate choice for a scalar function. A key feature of these formulations is an appearance of the nontrivial "dynamical" theory through breakdown of topological symmetry in the matrix model. It is closely examined why the nontrivial "dynamical" theory appears from the trivial topological field theory.

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1 Introduction

One of the most exciting achievements for theoretical physicists is surely to construct a theory that explains all the experimental data observed by then and predicts still unknown phenomena existing in nature by starting with few fundamental principles deductively. As a representative example of such theories, we are familiar with general relativity by Einstein [1]. Even if general relativity is built from only two basic principles, namely, equivalence principle and general coordinate invariance by help of Riemannian geometry, it has explained not only all the physical facts relevant to gravity but also predicted various remarkable things such as the gravitational redshift, the precession of planetary orbits and the bending of light as well as an existence of black holes [2].

It is nowadays widely expected that string theory [3] might be the final theory unifying all the interactions among elementary particles including the gravitational interaction. Then, it is a fascinating enterprise to try to construct string theory or M-theory [4] from few fundamental principles like general relativity. However, at the present stage it is a very pity that our understanding of the fundamental principles in string theory is far from complete. Actually, in string theory we have a good grasp of neither the principle corresponding to the equivalence principle nor the gauge symmetry corresponding to the general covariance in comparison with general relativity.

Recently we have had some remarkable progress in non-perturbative formulations of M-theory [5] and IIB superstring [6]. These studies have provided us with an important clue to understand the fundamental degrees of freedom at the short distance in a theory containing gravity, where D-particles in M-theory and D-instantons in IIB superstring constitute the fundamental building blocks for membrane and string, respectively. However, from these studies it seems to be difficult to get useful informations directly about the underlying fundamental principle and gauge symmetry behind M-theory and string theory.

On the other hand, in a quest of the fundamental principle of string theory, Yoneya has advocated, what we call, the space-time uncertainty principle of string with respect to the time interval and the spacial length, which has the form [7, 8]

$$\Delta T \Delta X \geq l_s^2, \quad (1)$$

where l_s denotes the string minimum length which is related to the Regge slope α' by $l_s = \sqrt{\alpha'}$. The space-time uncertainty principle (1) would produce an interesting physical picture that in string theory, maybe also in M-theory, space-time in itself is quantized at the short distance and the concept of space-time as a continuum manifold cannot be extrapolated beyond the fundamental string scale l_s . It is also important to point out that this principle seems to be consistent with the recent non-perturbative formulations of M-theory [5] and IIB superstring [6] where this principle is realized implicitly in the form of the noncommutative geometry. Moreover, in terms of the "conformal constraint" coming from the Schild action [9] and essentially expressing the space-time uncertainty principle (1), Yoneya [8] has constructed a IIB matrix model from which the IKKT model [6] can be induced as an effective theory for D-branes [10].

Being stimulated by Yoneya's works [7, 8], in a preliminary study [11] we have recently constructed a bosonic matrix model and shown that the equation of motion precisely describes a stronger form of the space-time uncertainty principle (1). A key idea of this construction is to start with the topological field theory [12], break this huge symmetry and then give rise to a nontrivial dynamical matrix theory whose moduli space is equal to a stronger form of the space-time uncertainty relation (1). The aim of this article is not only to present the full details of this preliminary study [11] but also to generalize the results obtained there to a supersymmetric case in order to build matrix models for IIB superstring. As is well known at the moment, the supersymmetry is an essential ingredient in the recent development of the non-perturbative matrix models [5, 6] since the D-particle and the D-instanton are the BPS states preserving half of the supersymmetry and the supersymmetry guarantees the cluster property of these states.

It should be emphasized that our goal in this paper is to explore a possibility of formulating a non-perturbative string theory from the first principles. As the first principles, we shall take the space-time uncertainty principle and the topological symmetry since the former principle describes a peculiar feature of string theory and seems to be consistent with thought experiments done until now. On the other hand, although the latter principle is still conjectural in string theory, it is very appealing from the following arguments. A string has an infinite number of states in the perturbative level in addition to various extended objects as solitonic excitations in the non-perturbative regime. Thus the local symmetry behind a string theory must be quite huge such that it controls so many states simultaneously without reference to their masslessness or massiveness. The topological symmetry is a maximum local symmetry so that it would be a strong candidate as such a huge local symmetry.

Frankly speaking, however, at present we have no idea whether these two basic principles are really deep principles like the equivalence principle and the general covariance in general relativity or are just useful technical tools for construction of matrix models. Incidentally, as for the topological symmetry, it would be worthwhile to point out that it has been already stated that the topological symmetry might be of critical importance in both string theory and quantum gravity in connection with the background independent formulation of string theory and the unbroken phase of quantum gravity [13].

The paper is organized as follows. In section 2 we briefly review Yoneya's works [8] which are relevant to the present study. Specific attention is paid to the "conformal" constraint and the space-time uncertainty principle. In section 3, we derive a stronger form of the space-time uncertainty principle from the topological field theory where the classical action is trivially zero. The key idea here is the breakdown of the topological symmetry in changing from the continuous field theory to the discrete matrix model. In section 4, we incorporate the spinors in the above theory and construct a new matrix model. If we require this theory to be invariant under $N = 2$ supersymmetric transformations in ten dimensions, it turns out that this new matrix model becomes the IKKT model or the Yoneya model for type IIB superstring. This choice is dependent on the form of a classical solution for a scalar function. The final section is devoted to discussions.

2 The conformal constraint and the space-time uncertainty principle

In this section, we review only a part of Yoneya's works relevant to later study (See [7, 8] for more details). Let us start with the Schild action [9] of a bosonic string. Then the Schild action has the form

$$S_{Schild} = -\frac{1}{2} \int d^2\xi \left[-\frac{1}{2\lambda^2} \frac{1}{e} \left(\varepsilon^{ab} \partial_a X^\mu \partial_b X^\nu \right)^2 + e \right], \quad (2)$$

where $X^\mu(\xi)$ ($\mu = 0, 1, \dots, D-1$) are space-time coordinates, $e(\xi)$ is a positive definite scalar density defined on the string world sheet parametrized by ξ^1 and ξ^2 , and $\lambda = 4\pi\alpha'$. Throughout this paper, we assume that the space-time metric takes the flat Minkowskian form defined as $\eta_{\mu\nu} = \text{diag}(- + + \dots +)$.

Taking the variation with respect to the auxiliary field $e(\xi)$, one obtains

$$e(\xi) = \frac{1}{\lambda} \sqrt{-\frac{1}{2} (\varepsilon^{ab} \partial_a X^\mu \partial_b X^\nu)^2}, \quad (3)$$

which is also rewritten to be

$$\lambda^2 = -\frac{1}{2} \{X^\mu, X^\nu\}^2, \quad (4)$$

where one has introduced the diffeomorphism invariant Poisson bracket defined as

$$\{X^\mu, X^\nu\} = \frac{1}{e(\xi)} \varepsilon^{ab} \partial_a X^\mu \partial_b X^\nu. \quad (5)$$

Then eliminating the auxiliary field $e(\xi)$ from (2) through (3) and using the identity

$$-\det \partial_a X \cdot \partial_b X = -\frac{1}{2} \left(\varepsilon^{ab} \partial_a X^\mu \partial_b X^\nu \right)^2, \quad (6)$$

the Schild action (2) becomes at least classically equivalent to the famous Nambu-Goto action S_{NG}

$$\begin{aligned} S_{Schild} &= -\frac{1}{\lambda} \int d^2\xi \sqrt{-\det \partial_a X \cdot \partial_b X}, \\ &= S_{NG}. \end{aligned} \quad (7)$$

In order to check that the "conformal" constraint (4) expresses half of the classical Virasoro conditions, it is convenient to use the Hamiltonian formalism [8]. If we denote the differentiation with respect to ξ^1 and ξ^2 by the dot and the prime, respectively, the canonical conjugate momenta to the X^μ are given by

$$P^\mu = \frac{1}{\lambda^2} \frac{1}{e} \left(\dot{X}^\mu X'^2 - X'^\mu \dot{X} \cdot X' \right), \quad (8)$$

from which we can obtain the conventional classical Virasoro constraints

$$P \cdot X' = 0, \quad (9)$$

$$P^2 + \frac{1}{\lambda^2} X'^2 = 0, \quad (10)$$

where the lapse constraint (10) is a consequence of the "conformal" constraint (4) while the shift constraint (9) comes from (8) trivially.

Let us clarify the physical implication of the "conformal" constraint (4). We are now familiar with the well-known relation between the Poisson bracket and the commutation relation in the large N matrix model:

$$\{A, B\} \longleftrightarrow [A, B]. \quad (11)$$

Then the "conformal" constraint (4) becomes

$$\lambda^2 = -\frac{1}{2} [X^\mu, X^\nu]^2. \quad (12)$$

Recalling that the target space-time metric is now assumed to be $\eta_{\mu\nu} = \text{diag}(- + + \dots +)$, Eq.(12) can be rewritten to be

$$[X^0, X^i]^2 - \lambda^2 = \frac{1}{2} [X^i, X^j]^2, \quad (13)$$

where the summation over the transverse indices i, j is taken. The right-handed side of Eq.(13) is a positive definite hermitian operator, so this equation implies the space-time uncertainty principle (1) under an appropriately defined expectation value [8]. In this respect it is interesting to note that the signature of the space-time must be not Euclidean but Minkowskian in order to get the space-time uncertainty principle (1) from (12). This point might give one justifiable reasoning to the problem of space-time signature [14].

As shown above, since a feature of the classical conformal invariance is contained in the space-time uncertainty principle, it is natural to postulate that non-perturbative string theory should be formulated on the basis of this principle [8]. Actually, Yoneya has derived such an action which has a close connection with the IKKT model [6]. His construction of the action is in itself quite interesting but a little ambiguous and ad hoc. In particular, it is unclear what underlying symmetry exists behind the space-time uncertainty principle. In the next section, we shall take a different path of thought where we start with a topological field theory, from which we will derive the space-time uncertainty principle proposed by Yoneya through the breakdown of the topological symmetry in the large N matrix model. This derivation might suggest that the topological symmetry is the underlying fundamental symmetry behind the space-time uncertainty principle of string theory.

3 A bosonic matrix model

In this section let us construct a bosonic matrix model which expresses an essential content of the space-time uncertainty principle. The preliminary report was given in the ref.[11]. Let us start by considering a topological theory [12] where the classical action is trivially zero but dependent on the fields $X^\mu(\xi)$ and $e(\xi)$ as follows:

$$S_c = S_c(X^\mu(\xi), e(\xi)) = 0. \quad (14)$$

The BRST transformations corresponding to the topological symmetry are given by

$$\begin{aligned} \delta_B X^\mu &= \alpha^\mu, \quad \delta_B \alpha^\mu = 0, \\ \delta_B e &= e \eta, \quad \delta_B \eta = 0, \\ \delta_B \bar{c} &= b, \quad \delta_B b = 0, \end{aligned} \quad (15)$$

where ψ^μ and η are ghosts, and \bar{c} and b are respectively an antighost and an auxiliary field. Note that these BRST transformations are obviously nilpotent. Also notice that the BRST transformation $\delta_B e$ shows the character as a scalar density of e .

The idea, then, is to fix partially the topological symmetry corresponding to $\delta_B e$ by introducing an appropriate covariant gauge condition. A conventional covariant and nonsingular gauge condition would be $e = 1$ but this gauge choice is not suitable for the present purpose since it makes difficult to pass to the large N matrix theory. Then it is easy to check that if we demand the space-time covariance almost the unique choice up to its polynomial forms is nothing but the "conformal" constraint (4). Of course, there is an ambiguity whether the fundamental parameter λ must be nonzero or not from the viewpoint of the IKKT matrix model. To the problem we have the following opinions. Firstly, nonzero λ is more general than zero λ . Secondly, suppose that we have fine-tuned λ to be zero at the outset. But renormalization usually introduces such a dimensionful quantity into the quantum theory so that it is natural to include nonzero λ in the gauge condition from the beginning. Consequently the quantum action defined as $S_b = \int d^2\xi e L_b$ becomes

$$\begin{aligned} L_b &= \frac{1}{e} \delta_B \left[\bar{c} \left\{ e \left(\frac{1}{2} \{X^\mu, X^\nu\}^2 + \lambda^2 \right) \right\} \right], \\ &= b \left(\frac{1}{2} \{X^\mu, X^\nu\}^2 + \lambda^2 \right) - \bar{c} \left(\eta \left(-\frac{1}{2} \{X^\mu, X^\nu\}^2 + \lambda^2 \right) + 2 \{X^\mu, X^\nu\} \{X^\mu, \alpha^\nu\} \right), \end{aligned} \quad (16)$$

where the BRST transformations (15) were used. Here for later convenience it is useful to redefine the auxiliary field b by $b + \bar{c} \eta$. Then L_b can be cast into a simpler form

$$L_b = b \left(\frac{1}{2} \{X^\mu, X^\nu\}^2 + \lambda^2 \right) - 2\lambda^2 \bar{c} \eta - 2\bar{c} \{X^\mu, X^\nu\} \{X^\mu, \alpha^\nu\}. \quad (17)$$

What is necessary to obtain a stronger form of the space-time uncertainty relation (12) is to change to the large N matrix theory where in addition to (11) we have the following

correspondence

$$\begin{aligned} \int d^2\xi \, e &\longleftrightarrow \text{Trace}, \\ \int De &\longleftrightarrow \sum_{n=1}^{\infty}, \end{aligned} \quad (18)$$

where the trace is taken over $SU(n)$ group. These correspondence can be justified by expanding the hermitian matrices by $SU(n)$ generators in the large N limit as is reviewed by the reference [15]. We will discuss this point in detail in the final section. Hence, for a moment, we assume that these correspondence is valid in our model.

Now in the large N limit, we have

$$S_b = \text{Tr} \left(b \left(\frac{1}{2} [X^\mu, X^\nu]^2 + \lambda^2 \right) - 2\lambda^2 \bar{c} \, \eta - 2\bar{c} [X^\mu, X^\nu] [X^\mu, \alpha^\nu] \right). \quad (19)$$

Then the partition function is defined as

$$\begin{aligned} Z &= \int DX^\mu D\alpha^\mu De D\eta D\bar{c} Db \, e^{-S_b}, \\ &= \sum_{n=1}^{\infty} \int DX^\mu D\alpha^\mu D\eta D\bar{c} Db \, e^{-S_b}. \end{aligned} \quad (20)$$

At this stage, it is straightforward to perform the path integration over η and \bar{c} . Consequently, one obtains

$$Z = \sum_{n=1}^{\infty} \int DX^\mu D\alpha^\mu Db \, e^{-\text{Tr} \, b \left(\frac{1}{2} [X^\mu, X^\nu]^2 + \lambda^2 \right)}. \quad (21)$$

In (21) since the quantum action does not depend on α^μ it is obvious that there remains the gauge symmetry

$$\delta\alpha^\mu = \omega^\mu, \quad (22)$$

which is of course nothing but the remaining topological symmetry. Now let us factor out this gauge volume or equivalently fix this gauge symmetry by the gauge condition $\alpha^\mu = 0$, so that the partition function is finally given by

$$Z = \sum_{n=1}^{\infty} \int DX^\mu Db \, e^{-\text{Tr} \, b \left(\frac{1}{2} [X^\mu, X^\nu]^2 + \lambda^2 \right)}. \quad (23)$$

It is remarkable that the variation of the action with respect to the auxiliary variable b in (23) gives a stronger form of the space-time uncertainty relation (12) and the theory is "dynamical" in the sense that the ghosts have completely been decoupled in (23). In other words, we have shown how to derive the space-time uncertainty principle from a topological theory through the breakdown of the topological symmetry in the large N matrix model.

Why has the topological theory yielded the nontrivial "dynamical" theory? The reason is very much simple. In changing from the continuous theory (17) to the matrix theory (19), the dynamical degree of freedom associated with $e(\xi)$ was replaced by the discrete sum over n while the corresponding BRST partner η remains the continuous variable. This distinct treatment of the BRST doublet leads to the breakdown of the topological symmetry giving rise to a "dynamical" matrix theory. In this respect, it is worthwhile to point out that while the topological symmetry is "spontaneously" broken in this process, the other gauge symmetries never be violated (Of course, correctly speaking, these gauge symmetries reduce to the global symmetries in the matrix model but this is irrelevant to the present argument). Moreover, notice that the above-examined phenomenon is a peculiar feature in the matrix model with the scalar density $e(\xi)$, which means that an existence of the gravitational degree of freedom is an essential ingredient since the generators of the world-sheet reparametrizations, the Virasoro operators, provide the Ward-identities associated with the target space general covariance.

4 Supersymmetric matrix models

Having obtained a bosonic matrix model, we now turn our attention to a more interesting model, i.e., its generalization to a supersymmetric matrix model. Actually, recent non-perturbative formulations of M-theory [5] and IIB superstring [6] are based on the supersymmetry. Here we should emphasize that our philosophy in constructing a supersymmetric matrix model is rather different from the attitude in the bosonic case in the previous section although we will go along a similar path of procedure in what follows. Namely, so far by starting with the topological field theory [12], we have tried to derive the space-time uncertainty principle proposed by Yoneya [7, 8]. In this section, we promote the space-time uncertainty principle to one of the basic principles for construction of a supersymmetric matrix model. In other words, as mentioned in the abstract and the introduction, on the basis of only two basic principles which are the space-time uncertainty principle of string and the topological symmetry, we attempt to construct a new supersymmetric matrix model. Of course, in the process of the model building, we will furthermore demand the invariance under the supersymmetric transformation. Although the topological symmetry is broken (in some case even the space-time uncertainty principle is not explicit) at the final stage, we will keep the strict invariance of a theory under the supersymmetry. In this sense, at the present stage our basic principles might be interpreted as the starting principles for the model building.

As a first step for constructing a supersymmetric matrix model, one has to require the classical action to depend on the Majorana spinor field $\psi_\alpha(\xi)$ as well as the bosonic fields $X^\mu(\xi)$ and $e(\xi)$

$$S_c = S_c(X^\mu(\xi), \psi_\alpha(\xi), e(\xi)) = 0, \quad (24)$$

where the subscript α stands for spinor index which should not be confused with the topological ghost $\alpha^\mu(\xi)$ corresponding to $X^\mu(\xi)$. The reason why we consider only the Majorana

spinor will be explained later. This time, in addition to the BRST transformations (15) one has to add the following BRST transformations for fermions:

$$\delta_B \psi_\alpha = \beta_\alpha, \quad \delta_B \beta_\alpha = 0. \quad (25)$$

Next let us set up the gauge condition for $\delta_B e$. Instead of the bosonic case

$$\frac{1}{2} \{X^\mu, X^\nu\}^2 + \lambda^2 = 0, \quad (26)$$

we shall set up its natural extension involving the spinor field

$$\frac{1}{2} \{X^\mu, X^\nu\}^2 + \lambda^2 + \frac{1}{2} \bar{\psi} \Gamma_\mu \{X^\mu, \psi\} = 0. \quad (27)$$

When transforming to the matrix theory later, this gauge condition becomes a generalized stronger form of the space-time uncertainty principle. Although this generalized form is different from the original one proposed by Yoneya [7, 8] by the spinor part, in the ground state they are equivalent so we take the above gauge condition (27). Interestingly enough, it will be shown later that the gauge choice (27) leads to the same theory as Yoneya's one if a suitable solution for the auxiliary variable is chosen. Incidentally, the spinor part in (27) is adopted from an analogy with the supersymmetric Yang-Mills theory. Thus we have the quantum action $S_q = \int d^2\xi \, e (L_b + L_f)$ with the bosonic contribution L_b (16) and the fermionic one L_f given by

$$\begin{aligned} L_f &= \frac{1}{e} \delta_B \left(\bar{c} \, e \, \frac{1}{2} \bar{\psi} \Gamma_\mu \{X^\mu, \psi\} \right), \\ &= b \, \frac{1}{2} \bar{\psi} \Gamma_\mu \{X^\mu, \psi\} - \bar{c} \, \frac{1}{2} \left(\bar{\beta} \Gamma_\mu \{X^\mu, \psi\} - \bar{\psi} \Gamma_\mu \{\alpha^\mu, \psi\} - \bar{\psi} \Gamma_\mu \{X^\mu, \beta\} \right). \end{aligned} \quad (28)$$

Here in a similar way to the bosonic case, let us redefine the auxiliary field b and the ghost β by $b + \bar{c} \, \eta$ and $\beta - \frac{1}{2} \psi \, \eta$, respectively. As a result, L_b is given by (17), on the other hand, L_f takes the same form as (28). When we rewrite the fermionic part L_f in this process, we need the famous Majorana identity $\bar{\psi} \Gamma_\mu \psi = 0$, for which we have confined ourselves to the Majorana spinor in this paper.

As before, at this stage let us pass to the matrix model. Again it is straightforward to carry out the path integration over \bar{c} and η in a perfect analogous way to the bosonic theory. Accordingly, we arrive at the following partition function

$$Z = \sum_{n=1}^{\infty} \int D X^\mu D \alpha^\mu D \psi_\alpha D \beta_\alpha D b \, e^{-Tr \, b \left(\frac{1}{2} [X^\mu, X^\nu]^2 + \lambda^2 + \frac{1}{2} \bar{\psi} \Gamma_\mu [X^\mu, \psi] \right)}. \quad (29)$$

In this expression since the quantum action is independent of α^μ and β_α we have the remaining topological symmetries given by

$$\delta \alpha^\mu = \omega^\mu, \quad \delta \beta_\alpha = \rho_\alpha. \quad (30)$$

After factoring these gauge volumes out, the partition function is finally cast to be

$$\begin{aligned} Z &= \sum_{n=1}^{\infty} \int DX^{\mu} D\psi_{\alpha} Db e^{-S_q}, \\ &= \sum_{n=1}^{\infty} \int DX^{\mu} D\psi_{\alpha} Db e^{-Tr b \left(\frac{1}{2}[X^{\mu}, X^{\nu}]^2 + \lambda^2 + \frac{1}{2}\bar{\psi}\Gamma_{\mu}[X^{\mu}, \psi] \right)}. \end{aligned} \quad (31)$$

Of course, the action S_q still possesses the zero volume reduction of the usual gauge symmetry

$$\begin{aligned} \delta\psi_{\alpha} &= i[X_{\mu}, \Lambda], \\ \delta X_{\mu} &= i[\psi, \Lambda], \\ \delta b &= i[b, \Lambda]. \end{aligned} \quad (32)$$

And it is straightforward to derive the equations of motion from S_q whose results are written as

$$\frac{1}{2}[X^{\mu}, X^{\nu}]^2 + \lambda^2 + \frac{1}{2}\bar{\psi}\Gamma_{\mu}[X^{\mu}, \psi] = 0, \quad (33)$$

$$[X^{\mu}, b[X_{\mu}, X_{\nu}]] + \frac{1}{4}[b\bar{\psi}\Gamma^{\nu}, \psi]_{+} = 0, \quad (34)$$

$$[X^{\mu}, \Gamma_{\mu}\psi]b + \frac{1}{2}\Gamma_{\mu}\psi[X^{\mu}, b] = 0, \quad (35)$$

where $[\ , \]_{+}$ denotes the anticommutator.

In this way, we have constructed a new matrix model with the Majorana spinor variable on the basis of the space-time uncertainty principle and the topological symmetry. Although the action contains the spinor variable in addition to the bosonic variable, it is not always supersymmetric. The supersymmetry plays the most critical role in the matrix models for M-theory [5] and IIB superstring theory [6], so we should require the invariance under the supersymmetry for the action S_q obtained in (31). The most natural form of $N = 2$ supersymmetric transformations is motivated by a supersymmetric Yang-Mills theory whose (0+0)-dimensional reduction is given by

$$\begin{aligned} \delta\psi_{\alpha}^{ab} &= i[X_{\mu}, X_{\nu}]^{ab}(\Gamma^{\mu\nu}\varepsilon)_{\alpha} + \zeta_{\alpha}\delta^{ab}, \\ \delta X_{\mu}^{ab} &= i\bar{\varepsilon}\Gamma_{\mu}\psi^{ab}, \\ \delta b^{ab} &= 0, \end{aligned} \quad (36)$$

where we have explicitly written down the matrix indices to clarify that ε_{α} and ζ_{α} are the Majorana spinor parameters. These supersymmetric transformations are of the same form as in IKKT model [6]. At this stage, we assume the space-time dimensions to be ten in order to make contact with IIB superstring.

To make the action S_q in (31) invariant under the $N = 2$ supersymmetry (36), it is easy to check that b^{ab} must take the diagonal form with respect to the hermitian matrix indices. There are two interesting solutions. One of them is to select the auxiliary variable b^{ab} to be

proportional to δ^{ab} up to a constant. Without generality we take the proportional constant to be $-\frac{1}{2}$, therefore

$$b^{ab} = -\frac{1}{2}\delta^{ab}. \quad (37)$$

Here if we redefine X^μ , ψ , and $-\frac{1}{2}\lambda^2$ in terms of $\alpha^{\frac{1}{4}}X^\mu$, $\sqrt{2}\alpha^{\frac{3}{8}}\psi$, and β , respectively, the action S_q can be rewritten to be

$$S_q = \alpha \left(-\frac{1}{4} \text{Tr} [X^\mu, X^\nu]^2 - \frac{1}{2} \text{Tr} \bar{\psi} \Gamma_\mu [X^\mu, \psi] \right) + \beta \text{Tr} \mathbf{1}. \quad (38)$$

Note that this action is completely equivalent to the action in the IKKT model [6]. In this case, we cannot derive the space-time uncertainty relation from the equation of motion, but this relation might be encoded implicitly in the matrix character of the model.

The other interesting solution would be of the form

$$b^{ab} = c \delta^{ab}, \quad (39)$$

with some additional auxiliary variable c . With this choice, the partition function (31) can be reduced to be

$$\begin{aligned} Z &= \sum_{n=1}^{\infty} \int DX^\mu D\psi_\alpha Dc e^{-S_q}, \\ &= \sum_{n=1}^{\infty} \int DX^\mu D\psi_\alpha Dc e^{-c \text{Tr} \left(\frac{1}{2} [X^\mu, X^\nu]^2 + \lambda^2 + \frac{1}{2} \bar{\psi} \Gamma_\mu [X^\mu, \psi] \right)}. \end{aligned} \quad (40)$$

At first sight, it seems that we have obtained a new supersymmetric matrix model, but this is an illusion. We shall show that the above model is entirely equivalent to the Yoneya model [8] in what follows. Provided that we take account of the stronger form of the space-time uncertainty principle instead of the weaker form, the Yoneya model can be expressed in terms of the partition function

$$\begin{aligned} Z &= \sum_{n=1}^{\infty} \int DX^\mu D\psi_\alpha Dc e^{-S_y}, \\ &= \sum_{n=1}^{\infty} \int DX^\mu D\psi_\alpha Dc e^{-c \text{Tr} \left(\frac{1}{2} [X^\mu, X^\nu]^2 + \lambda^2 \right) - \text{Tr} \frac{1}{2} \bar{\psi} \Gamma_\mu [X^\mu, \psi]}. \end{aligned} \quad (41)$$

This partition in the Yoneya model does not look like the partition (40). But Yoneya has defined the supersymmetric transformations in a slightly different manner compared to ours (36). His supersymmetry is

$$\begin{aligned} \delta \psi_\alpha^{ab} &= ic [X_\mu, X_\nu]^{ab} (\Gamma^{\mu\nu} \varepsilon)_\alpha + \zeta_\alpha \delta^{ab}, \\ \delta X_\mu^{ab} &= i \bar{\varepsilon} \Gamma_\mu \psi^{ab}, \\ \delta c &= 0. \end{aligned} \quad (42)$$

Note that there exists c variable in the first term of the right-handed side in the first equation while it is absent in our formula (36) (Of course, in (36) we should replace $\delta b^{ab} = 0$ with $\delta c = 0$ for present consideration). Then it is easy to show that if we redefine ψ , ε and ζ by $c^{\frac{1}{2}}\psi$, $c^{-\frac{1}{2}}\varepsilon$ and $c^{\frac{1}{2}}\zeta$, respectively in the Yoneya model, Yoneya's action S_y and supersymmetric transformations (42) conform to our action S_q and supersymmetric transformations (36), respectively. To demonstrate a complete equivalence, we have to consider the functional measure. From these redefinitions the functional measure receives a contribution of an additional factor c^8 , but this change is absorbed into a definition of the functional measure Dc since the variable c is the supersymmetrically invariant non-dynamical auxiliary variable in the model at hand. In this way, we can show that the solution (39) gives rise to the Yoneya model. It is surprising that depending on a choice of the scalar function b our model leads to the IKKT model [6] and the Yoneya model [8], which on reflection clarifies the difference between both the matrix models.

5 Discussions

In this article we have investigated mainly two problems. One of them is a possibility of the space-time uncertainty principle advocated by Yoneya [7, 8] to be derived from the topological field theory [11]. This study suggests that the underlying symmetry behind this principle in string theory might be a topological symmetry as mentioned before in a different context [13]. The other problem is to derive the supersymmetric matrix models from the first principles based on the space-time uncertainty principle and the topological symmetry and examine the relation between the matrix model obtained in this way and the known matrix models. We have observed that our matrix model contains both the IKKT model and the Yoneya model if we demand the supersymmetry.

A rather unexpected appearance of the topological field theory seems to be plausible from the following intuitive arguments. Suppose that we live in the world where the topological symmetry is exactly valid. In such a world we have no means of measuring any distance owing to lack of the metric tensor field so that there is neither concept of distance nor the space-time uncertainty principle. But once the topological symmetry which is particularly connected with the gravitational degrees of freedom, is spontaneously broken by some dynamical mechanism, an existence of the dynamical metric together with a string having the minimum length would give us both concepts of the distance and the space-time uncertainty principle. Our bosonic matrix model seems to realize this scenario in a concrete way.

Here we would like to comment on one important problem. In our models, as in the IKKT model [6] the matrix size n is now regarded as a dynamical variable so that the partition function includes the summation over n . Even if the direct proof is missing, the summation over n is expected to recover the path integration over $e(\xi)$. In fact, the authors of the reference [16] have recently shown that the model of Fayyazuddin et al. [15, 17] where a positive definite hermitian matrix Y is introduced as a dynamical variable instead of n , belongs to the

same universality class as the IKKT model [6] owing to irrelevant deformations of the loop equation [18]. Thus we think that the correspondence (12) and (18) are legitimate even in the context at hand. Related to this problem, there is an interesting recent conjecture in the non-perturbative formulation of M-theory that the equivalence between M-theory and Matrix theory is not limited to the large N limit but is valid for finite N [19]. More recently this conjecture has been proved to be correct up to two loops by evaluating the effective action for the scattering of two D0-branes [20, 21].

In the bosonic model, we have not paid attention to the number of the space-time dimensions. In fact any dimensions except $D < 2$ are allowed. But an intriguing case happens when $D = 2$ even if this specification is not always necessary within the formulation. In this special dimension, the Nambu-Goto action which is at least classically equivalent to the Schild action as shown in (7) becomes not only the topological field theory but also almost a surface term as follows:

$$\begin{aligned}\sqrt{-\det \partial_a X \cdot \partial_b X} &= \sqrt{-(\det \partial_a X^\mu)^2}, \\ &= \pm \det \partial_a X^\mu, \\ &= \mp \frac{1}{2} \varepsilon^{ab} \varepsilon_{\mu\nu} \partial_a X^\mu \partial_b X^\nu,\end{aligned}\tag{43}$$

where we have assumed a smooth parametrization of X^μ over ξ^a in order to take out the absolute value. Actually, this topological model has been investigated to some extent in the past [22, 23, 24]. In this case it is interesting that we can start with not zero but the nontrivial surface term as a classical action.

Our approach heavily relies on the mechanism of the breakdown of the topological symmetry, so we should examine more closely the reason why our model gives rise to the nontrivial "dynamical" theory from at least classically trivial topological theory. As mentioned in section 3, the technical reason lies in asymmetric treatment between the BRST doublet e and η . However, there exists a deeper reason behind it. To make our arguments clear, it is useful to compare the present approach with the previous studies about the topological (pregauge-) pregeometric models [23, 24] whose essential ideas will be recapitulated in what follows.

For generality, we consider an arbitrary dimension of space-time. We take the Nambu-Goto action as a classical action where we restrict ourselves to the case that the dimension is equal between the world-volume and the space-time. Then in a similar argument to (43) we can prove that this classical action becomes topological. This is because we can eliminate all the dynamical degrees of freedom by means of the world-volume reparametrizations. Let us rewrite it to the Polyakov form

$$\begin{aligned}S &= -\frac{1}{\lambda} \int d^D \xi \sqrt{-\det \partial_a X \cdot \partial_b X}, \\ &= \int d^D \xi \sqrt{-g} (g^{ab} \partial_a X^\mu \partial_b X^\nu + \lambda).\end{aligned}\tag{44}$$

In spite of lack of proof, the above two actions might be equivalent even in the quantum level as well as the classical level owing to the topological character where there is no anomaly.

Next work is to evaluate the effective action for the metric g^{ab} due to the quantum fluctuation of the "matter" fields X^μ whose result is given by [25]

$$S_{eff} = i \text{Tr} \log \left[\left(\partial_a \sqrt{-g} g^{ab} \partial_b \right) \right] + \lambda \int d^D \xi \sqrt{-g}. \quad (45)$$

When the curvature is small, it reduces to the Einstein-Hilbert action with the cosmological constant

$$S_{eff} = \int d^D \xi \sqrt{-g} \left(\tilde{\lambda} + \frac{1}{16\pi G} R + O(R^2, \log \Lambda^2) \right), \quad (46)$$

with

$$\begin{aligned} \tilde{\lambda} &= \frac{D\Lambda^4}{8(4\pi)^2} + \lambda, \\ \frac{1}{16\pi G} &= \frac{D\Lambda^2}{24(4\pi)^2}, \end{aligned} \quad (47)$$

where we have introduced the momentum cutoff Λ of the Pauli-Villars type. Note that (47) shows that we can choose the effective cosmological constant $\tilde{\lambda}$ as small as we want, and the cutoff Λ is of the order the Planck mass. It is quite interesting to ask why the topological action has produced the Einstein-Hilbert action. This is because the momentum cutoff Λ breaks the topological symmetry with keeping the general covariance. In other words, we have secretly introduced seed for breaking the topological symmetry by the form of the cutoff. Of course, it is an interesting idea to make a conjecture that renormalization induces such a scale, but it seems to be quite difficult to prove this conjecture.

From this point of view, it is valuable to reconsider why the present formulation has produced the nontrivial matrix models from the topological field theory. Originally, in membrane world, the matrix model has appeared to regularize the lightcone supermembrane action with area-preserving diffeomorphisms where it has been remarkably shown that the action becomes exactly that of ten dimensional $SU(n)$ supersymmetric Yang-Mills theory reduced to $(0+1)$ -dimensions [26]. Similarly, in our models, changing from the continuous topological field theory to the discrete matrix model is equal to an introduction of the regularization where the regularization parameter corresponds to the size of the matrices. This type of the regularization breaks only the topological symmetry, from which we can obtain the nontrivial "dynamical" matrix models. It is very interesting that the matrix model is equipped with such a natural regularization scheme in itself. If the topological symmetry is truly broken by some mechanism in order to make the topological field theory a physically vital theory, we believe that theories equipped with some natural regularization scheme such as matrix model and induced gravity (pregeometry) would play an important role. In connection with string theory with $\frac{1}{N}$ expansion, it is remarkable that several years ago Thorn has already made a conjecture that the local theory underlying string theory should be either a theory with no curvature terms, as in induced gravity or a topological field theory [27]. The present formalism realizes this conjecture to a certain extent.

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